

Mathematica 11.3 Integration Test Results

Test results for the 284 problems in "Hearn Problems.m"

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+x^2+x^4} dx$$

Optimal (type 3, 67 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{2\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{2\sqrt{3}} - \frac{1}{4} \text{Log}[1-x+x^2] + \frac{1}{4} \text{Log}[1+x+x^2]$$

Result (type 3, 73 leaves):

$$\frac{i \left(\sqrt{1-i\sqrt{3}} \text{ArcTan}\left[\frac{1}{2}(-i+\sqrt{3})x\right] - \sqrt{1+i\sqrt{3}} \text{ArcTan}\left[\frac{1}{2}(i+\sqrt{3})x\right] \right)}{\sqrt{6}}$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{2+x^2+x^4} dx$$

Optimal (type 3, 196 leaves, 9 steps):

$$-\frac{1}{2} \sqrt{\frac{1}{14}(-1+2\sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right] +$$

$$\frac{1}{2} \sqrt{\frac{1}{14}(-1+2\sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right] - \frac{\text{Log}[\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2]}{4\sqrt{2(-1+2\sqrt{2})}} + \frac{\text{Log}[\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2]}{4\sqrt{2(-1+2\sqrt{2})}}$$

Result (type 3, 91 leaves):

$$- \frac{i \operatorname{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right]}{\sqrt{\frac{7}{2}(1-i\sqrt{7})}} + \frac{i \operatorname{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right]}{\sqrt{\frac{7}{2}(1+i\sqrt{7})}}$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{2-x^2+x^4} dx$$

Optimal (type 3, 196 leaves, 9 steps):

$$- \frac{1}{2} \sqrt{\frac{1}{14}(1+2\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right] +$$

$$\frac{1}{2} \sqrt{\frac{1}{14}(1+2\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right] - \frac{\operatorname{Log}\left[\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2\right]}{4\sqrt{2(1+2\sqrt{2})}} + \frac{\operatorname{Log}\left[\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2\right]}{4\sqrt{2(1+2\sqrt{2})}}$$

Result (type 3, 91 leaves):

$$- \frac{i \operatorname{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}(-1-i\sqrt{7})}}\right]}{\sqrt{\frac{7}{2}(-1-i\sqrt{7})}} + \frac{i \operatorname{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}(-1+i\sqrt{7})}}\right]}{\sqrt{\frac{7}{2}(-1+i\sqrt{7})}}$$

Problem 51: Result is not expressed in closed-form.

$$\int \frac{1}{1-x^4+x^8} dx$$

Optimal (type 3, 275 leaves, 19 steps):

$$- \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}} - \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right]}{2\sqrt{6}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right]}{2\sqrt{6}} -$$

$$\frac{\operatorname{Log}\left[1-\sqrt{2-\sqrt{3}}x+x^2\right]}{4\sqrt{6}} + \frac{\operatorname{Log}\left[1+\sqrt{2-\sqrt{3}}x+x^2\right]}{4\sqrt{6}} - \frac{\operatorname{Log}\left[1-\sqrt{2+\sqrt{3}}x+x^2\right]}{4\sqrt{6}} + \frac{\operatorname{Log}\left[1+\sqrt{2+\sqrt{3}}x+x^2\right]}{4\sqrt{6}}$$

Result (type 7, 42 leaves):

$$\frac{1}{4} \text{RootSum}\left[1 - \#1^4 + \#1^8 \ \&, \frac{\text{Log}[x - \#1]}{-\#1^3 + 2 \#1^7} \ \&\right]$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int \frac{x^7}{1+x^{12}} dx$$

Optimal (type 3, 49 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x^4}{\sqrt{3}}\right]}{4\sqrt{3}} - \frac{1}{12} \text{Log}[1+x^4] + \frac{1}{24} \text{Log}[1-x^4+x^8]$$

Result (type 3, 260 leaves):

$$\begin{aligned} & \frac{1}{24} \left(2\sqrt{3} \text{ArcTan}\left[\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right] - 2\sqrt{3} \text{ArcTan}\left[\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right] + \right. \\ & \quad \left. 2\sqrt{3} \text{ArcTan}\left[\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right] - 2\sqrt{3} \text{ArcTan}\left[\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right] - 2 \text{Log}[1-\sqrt{2}x+x^2] - 2 \text{Log}[1+\sqrt{2}x+x^2] + \right. \\ & \quad \left. \text{Log}[2+\sqrt{2}x-\sqrt{6}x+2x^2] + \text{Log}[2+\sqrt{2}(-1+\sqrt{3})x+2x^2] + \text{Log}[2-(\sqrt{2}+\sqrt{6})x+2x^2] + \text{Log}[2+(\sqrt{2}+\sqrt{6})x+2x^2] \right) \end{aligned}$$

Problem 81: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[x] dx$$

Optimal (type 3, 3 leaves, 1 step):

$$\text{ArcTanh}[\text{Sin}[x]]$$

Result (type 3, 33 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[x] dx$$

Optimal (type 3, 5 leaves, 1 step):

$$-\text{ArcTanh}[\text{Cos}[x]]$$

Result (type 3, 17 leaves):

$$-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \text{Cos}[a + b x] dx$$

Optimal (type 3, 10 leaves, 1 step):

$$\frac{\text{Sin}[a + b x]}{b}$$

Result (type 3, 21 leaves):

$$\frac{\text{Cos}[b x] \text{Sin}[a]}{b} + \frac{\text{Cos}[a] \text{Sin}[b x]}{b}$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[a + b x] dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{\text{ArcTanh}[\text{Cos}[a + b x]]}{b}$$

Result (type 3, 38 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{b x}{2}\right]\right]}{b} + \frac{\text{Log}\left[\text{Sin}\left[\frac{a}{2} + \frac{b x}{2}\right]\right]}{b}$$

Problem 112: Result more than twice size of optimal antiderivative.

$$\int \text{Sec}[a + b x] dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{ArcTanh}[\text{Sin}[a + b x]]}{b}$$

Result (type 3, 68 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] - \text{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b} + \frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2} + \frac{bx}{2}\right] + \text{Sin}\left[\frac{a}{2} + \frac{bx}{2}\right]\right]}{b}$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 + \text{Sin}[x]} dx$$

Optimal (type 3, 10 leaves, 1 step):

$$-\frac{\text{Cos}[x]}{1 + \text{Sin}[x]}$$

Result (type 3, 23 leaves):

$$\frac{2 \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - \text{Sin}[x]} dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{Cos}[x]}{1 - \text{Sin}[x]}$$

Result (type 3, 25 leaves):

$$\frac{2 \text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]}$$

Problem 190: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1 + x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Result (type 3, 38 leaves):

$$-\frac{1}{2} \text{Log}\left[1 - \frac{x}{\sqrt{-1+x^2}}\right] + \frac{1}{2} \text{Log}\left[1 + \frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 197: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x \sqrt{-1+x^2-x^4}} dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\frac{1}{2} \text{ArcTan}\left[\frac{2-x^2}{2\sqrt{-1+x^2-x^4}}\right]$$

Result (type 3, 37 leaves):

$$-i \text{Log}[x] + \frac{1}{2} i \text{Log}\left[-2+x^2+2i\sqrt{-1+x^2-x^4}\right]$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int \left(\frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx$$

Optimal (type 3, 27 leaves, 5 steps):

$$10 \text{ArcTanh}\left[\frac{x}{\sqrt{-4+x^2}}\right] + \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Result (type 3, 71 leaves):

$$-5 \text{Log}\left[1 - \frac{x}{\sqrt{-4+x^2}}\right] + 5 \text{Log}\left[1 + \frac{x}{\sqrt{-4+x^2}}\right] - \frac{1}{2} \text{Log}\left[1 - \frac{x}{\sqrt{-1+x^2}}\right] + \frac{1}{2} \text{Log}\left[1 + \frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right]}{\sqrt{\alpha^2 + \epsilon^2}}$$

Result (type 3, 58 leaves):

$$-\frac{i \text{Log}\left[\frac{2\left(-i\sqrt{\alpha^2 + \epsilon^2} + \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}\right)}{r}\right]}{\sqrt{\alpha^2 + \epsilon^2}}$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr$$

Optimal (type 3, 37 leaves, 2 steps):

$$-\frac{\text{ArcTan}\left[\frac{\alpha^2 + kr}{\alpha\sqrt{-\alpha^2 - 2kr + 2hr^2}}\right]}{\alpha}$$

Result (type 3, 48 leaves):

$$-\frac{i \text{Log}\left[\frac{2\left(-\frac{i(\alpha^2 + kr)}{\alpha} + \sqrt{-\alpha^2 - 2kr + 2hr^2}\right)}{r}\right]}{\alpha}$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{\text{ArcTan}\left[\frac{\alpha^2 + \epsilon^2 + kr}{\sqrt{\alpha^2 + \epsilon^2}\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}}\right]}{\sqrt{\alpha^2 + \epsilon^2}}$$

Result (type 3, 72 leaves):

$$-\frac{i \operatorname{Log} \left[\frac{2 \left(-\frac{i (\alpha^2 + \epsilon^2 + k r)}{\sqrt{\alpha^2 + \epsilon^2}} + \sqrt{-\alpha^2 - \epsilon^2 + 2 r (-k + h r)} \right)}{r} \right]}{\sqrt{\alpha^2 + \epsilon^2}}$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2 e r^2 - 2 k r^4}} dr$$

Optimal (type 3, 56 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan} \left[\frac{e - 2 k r^2}{\sqrt{2} \sqrt{k} \sqrt{-\alpha^2 + 2 e r^2 - 2 k r^4}} \right]}{2 \sqrt{2} \sqrt{k}}$$

Result (type 3, 66 leaves):

$$\frac{i \operatorname{Log} \left[-\frac{i \sqrt{2} (-e + 2 k r^2)}{\sqrt{k}} + 2 \sqrt{-\alpha^2 + 2 e r^2 - 2 k r^4} \right]}{2 \sqrt{2} \sqrt{k}}$$

Problem 213: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r \sqrt{-\alpha^2 + 2 h r^2 - 2 k r^4}} dr$$

Optimal (type 3, 44 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan} \left[\frac{\alpha^2 - h r^2}{\alpha \sqrt{-\alpha^2 + 2 h r^2 - 2 k r^4}} \right]}{2 \alpha}$$

Result (type 3, 59 leaves):

$$-\frac{i \operatorname{Log} \left[\frac{-2 i \alpha^2 + 2 i h r^2 + 2 \alpha \sqrt{-\alpha^2 + 2 r^2 (h - k r^2)}}{\alpha r^2} \right]}{2 \alpha}$$

Problem 214: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{r \sqrt{-\alpha^2 - \epsilon^2 + 2 h r^2 - 2 k r^4}} dr$$

Optimal (type 3, 68 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\alpha^2 + \epsilon^2 - h r^2}{\sqrt{\alpha^2 + \epsilon^2} \sqrt{-\alpha^2 - \epsilon^2 + 2 h r^2 - 2 k r^4}}\right]}{2 \sqrt{\alpha^2 + \epsilon^2}}$$

Result (type 3, 80 leaves):

$$-\frac{i \text{Log}\left[\frac{2 \left(-\frac{i(\alpha^2 + \epsilon^2 - h r^2)}{\sqrt{\alpha^2 + \epsilon^2}} + \sqrt{-\alpha^2 - \epsilon^2 + 2 h r^2 - 2 k r^4} \right)}{r^2}\right]}{2 \sqrt{\alpha^2 + \epsilon^2}}$$

Problem 235: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 + \sin[x]} dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{2 \cos[x]}{\sqrt{1 + \sin[x]}}$$

Result (type 3, 40 leaves):

$$\frac{2 \left(-\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right) \sqrt{1 + \sin[x]}}{\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]}$$

Problem 236: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 - \sin[x]} dx$$

Optimal (type 3, 14 leaves, 1 step):

$$\frac{2 \cos[x]}{\sqrt{1 - \sin[x]}}$$

Result (type 3, 42 leaves):

$$\frac{2 \left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right) \sqrt{1 - \sin[x]}}{\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}$$

Problem 256: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{-1 + x^4} dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[x^2]$$

Result (type 3, 23 leaves):

$$\frac{1}{4} \operatorname{Log}[1 - x^2] - \frac{1}{4} \operatorname{Log}[1 + x^2]$$

Problem 278: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

Optimal (type 3, 94 leaves, ? steps):

$$\frac{(1 + 2x) \sqrt{1 + 2x^2 + 4x^3 + x^4}}{2(-1 + 2x^2)} - \operatorname{ArcTanh}\left[\frac{x(2 + x)(7 - x + 27x^2 + 33x^3)}{(2 + 37x^2 + 31x^3) \sqrt{1 + 2x^2 + 4x^3 + x^4}}\right]$$

Result (type 4, 5137 leaves):

$$\frac{(1 + 2x) \sqrt{1 + 2x^2 + 4x^3 + x^4}}{2(-1 + 2x^2)} + \left(5 \left(x - \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 1\right] \right)^2 \right. \\ \left. \left(\left(1 + \frac{1}{\sqrt{2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(1 + x) \left(\operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 1\right] - \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 3\right]\right)}{\left(x - \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 3\right]\right)}\right]}{\left(\left(\operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 1\right] - \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 2\right]\right) \left(1 + \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 3\right]\right) \right) / \left(\left(1 + \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 2\right]\right) \left(\operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 1\right] - \operatorname{Root}\left[1 - \#1 + 3\#1^2 + \#1^3 \&, 3\right]\right)} \right) \right) -$$

$$\begin{aligned}
& \text{EllipticPi} \left[\frac{\left(-\frac{1}{\sqrt{2}} + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]\right) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{\left(-1 - \frac{1}{\sqrt{2}}\right) \left(-\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}, \right. \\
& \left. \text{ArcSin} \left[\sqrt{-\frac{(1+x) \left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}} \right], \right. \\
& \left. \left(\frac{\left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]\right) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{\left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]\right) \left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)} \right) / \right. \\
& \left. \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]\right) \right) \sqrt{\frac{(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]\right)}} \\
& \sqrt{\frac{(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}} \\
& \sqrt{-\frac{(1+x) \left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}} \\
& \left. \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right) \right) / \\
& \left(2 \left(-1 - \frac{1}{\sqrt{2}}\right) \sqrt{1 + 2x^2 + 4x^3 + x^4} \left(\frac{1}{\sqrt{2}} - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]\right) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]\right) \right. \\
& \left. \left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right) \right) + \\
& \left(5 \sqrt{2} (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1])^2 \right. \\
& \left. \left(\left(1 + \frac{1}{\sqrt{2}}\right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{(1+x) \left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}} \right], \right. \right. \\
& \left. \left(\frac{\left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]\right) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{\left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]\right) \left(\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)} \right) / \right. \\
& \left. \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]\right) \right) \right) - \\
& \text{EllipticPi} \left[\frac{\left(-\frac{1}{\sqrt{2}} + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]\right) \left(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}{\left(-1 - \frac{1}{\sqrt{2}}\right) \left(-\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]\right)}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \sqrt{\frac{\left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right)}{\left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right)}} \right. \\
& \sqrt{\frac{\left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}{\left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}} \\
& \sqrt{-\frac{(1+x) \left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}{\left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}} \\
& \left. \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right) \right) / \\
& \left(2 \left(-1 + \frac{1}{\sqrt{2}} \right) \sqrt{1 + 2x^2 + 4x^3 + x^4} \left(-\frac{1}{\sqrt{2}} - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] \right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \right. \\
& \left. \left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right] \right) \right) - \\
& \left(5 \sqrt{2} \left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right)^2 \right. \\
& \left. \left(\left(1 - \frac{1}{\sqrt{2}} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(1+x) \left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}{\left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}} \right], \right. \\
& \left. \frac{\left(\left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)\right) /}{\left(\left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right) \left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)\right)} \right] - \\
& \text{EllipticPi}\left[\frac{\left(\frac{1}{\sqrt{2}} + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}{\left(-1 + \frac{1}{\sqrt{2}}\right) \left(-\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)} \right], \\
& \text{ArcSin}\left[\sqrt{-\frac{(1+x) \left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}{\left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right)}} \right], \\
& \left(\left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right) \right) / \\
& \left(\left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right) \left(\text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right] - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3\right]\right) \right) \\
& \left. \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \right) \sqrt{\frac{\left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right)}{\left(x - \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1\right]\right) \left(1 + \text{Root}\left[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}} \\
& \sqrt{\frac{(1 + x) (\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}} \\
& (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]) \Bigg) / \\
& \left(\left(-1 + \frac{1}{\sqrt{2}} \right) \sqrt{1 + 2x^2 + 4x^3 + x^4} \left(-\frac{1}{\sqrt{2}} - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] \right) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) \right. \\
& \left. (\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]) \right) + \\
& \left(6 \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{(1 + x) (-\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}} \right], \right. \\
& \left. \left((\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]) (-1 - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]) \right) / \right. \\
& \left. \left((-1 - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2]) (\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]) \right) \right] \\
& (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1])^2 \sqrt{\frac{(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 2])}} \\
& (-1 - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]) \\
& \sqrt{\frac{(1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}} \\
& \sqrt{\frac{(1 + x) (-\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}{(x - \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3])}} \Bigg) / \\
& \left(\sqrt{1 + 2x^2 + 4x^3 + x^4} (1 + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1]) (-\text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 1] + \text{Root}[1 - \#1 + 3 \#1^2 + \#1^3 \&, 3]) \right)
\end{aligned}$$

Problem 279: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1+2y) \sqrt{1-5y-5y^2}}{y(1+y)(2+y) \sqrt{1-y-y^2}} dy$$

Optimal (type 3, 142 leaves, ? steps):

$$-\frac{1}{4} \operatorname{ArcTanh} \left[\frac{(1-3y) \sqrt{1-5y-5y^2}}{(1-5y) \sqrt{1-y-y^2}} \right] - \frac{1}{2} \operatorname{ArcTanh} \left[\frac{(4+3y) \sqrt{1-5y-5y^2}}{(6+5y) \sqrt{1-y-y^2}} \right] + \frac{9}{4} \operatorname{ArcTanh} \left[\frac{(11+7y) \sqrt{1-5y-5y^2}}{3(7+5y) \sqrt{1-y-y^2}} \right]$$

Result (type 4, 630 leaves):

$$\frac{1}{16 \sqrt{1-5y-5y^2} \sqrt{1-y-y^2}} \left(-1 - \frac{2}{\sqrt{5}} \right) (1+\sqrt{5}+2y)^2 \sqrt{\frac{5+3\sqrt{5}+10y}{5+5\sqrt{5}+10y}}$$

$$\left(20 \left(-4 \sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}} \sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}} + \sqrt{5} \sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}} \sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}} + 5 \sqrt{\frac{-5+\sqrt{5}+2\sqrt{5}y}{1+\sqrt{5}+2y}} \sqrt{\frac{-3+\sqrt{5}+2\sqrt{5}y}{1+\sqrt{5}+2y}} \right. \right.$$

$$\left. \left. 2\sqrt{5} \sqrt{\frac{-5+\sqrt{5}+2\sqrt{5}y}{1+\sqrt{5}+2y}} \sqrt{\frac{-3+\sqrt{5}+2\sqrt{5}y}{1+\sqrt{5}+2y}} \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{2 \sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}} \right], \frac{15}{16} \right] +$$

$$\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}} \sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}} \left(9\sqrt{5} \operatorname{EllipticPi} \left[\frac{5}{8} - \frac{\sqrt{5}}{8}, \operatorname{ArcSin} \left[\frac{2 \sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}} \right], \frac{15}{16} \right] + (-20+9\sqrt{5}) \right.$$

$$\left. \left. \operatorname{EllipticPi} \left[-\frac{3}{8} (-5+\sqrt{5}), \operatorname{ArcSin} \left[\frac{2 \sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}} \right], \frac{15}{16} \right] + 2\sqrt{5} \operatorname{EllipticPi} \left[\frac{3}{8} (5+\sqrt{5}), \operatorname{ArcSin} \left[\frac{2 \sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}} \right], \frac{15}{16} \right] \right) \right)$$

Problem 280: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(-\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left(1+\sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx$$

Optimal (type 3, 21 leaves, 1 step):

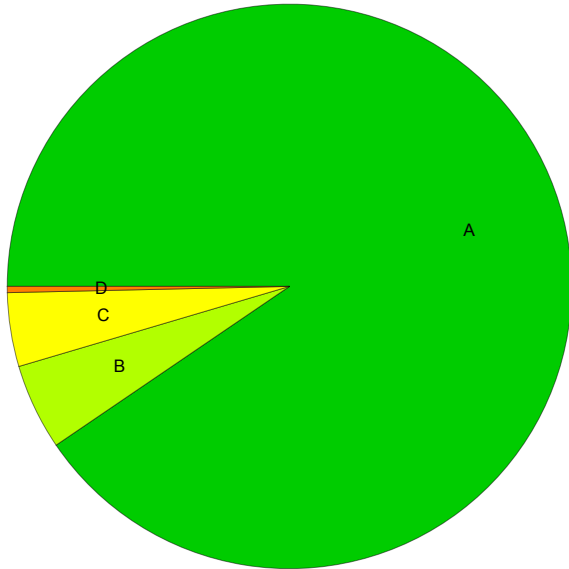
$$\text{Log}\left[1 + \sqrt{-4 + x^2} + \sqrt{-1 + x^2}\right]$$

Result (type 3, 97 leaves):

$$-\frac{1}{2} \text{ArcTanh}\left[\sqrt{-4 + x^2}\right] + \frac{1}{2} \text{ArcTanh}\left[\frac{1}{2} \sqrt{-1 + x^2}\right] + \frac{1}{4} \text{Log}\left[17 - 5x^2 - 4\sqrt{-4 + x^2}\sqrt{-1 + x^2}\right] + \frac{1}{4} \text{Log}\left[5 - 2x^2 - 2\sqrt{-4 + x^2}\sqrt{-1 + x^2}\right]$$

Summary of Integration Test Results

284 integration problems



A - 257 optimal antiderivatives

B - 14 more than twice size of optimal antiderivatives

C - 12 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts